

Algorithm Of The Distribution Geo / G / 1 / K / N Queue With Services Network

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Abstract: This paper represents an analysis of the Geo / G / 1 / K / N system in which the capacity of the service facility is less than N the total quantity of messages in the system. Consider the Geo / G / 1 / K / N system without a server holiday, assuming a late-arrival model with partial acceptance and a belated substitution rule, then a Geo / G / 1 / K / N Consider n systems with multiple servers. The service arrives at the end of each slot with a message probability in the source.

Keyword: Service facility, joint probability, elapsed vacation time, boundary,

1. INTRODUCTION:

Probability $\lambda(1 - \lambda)^{(\ell-1)}$ for each message $\ell = 1, 2, \dots$ remains in the source for ℓ slots before coming with which is a geometric distribution with a mean of $1/\lambda$ distribution slots. An incoming message is usually accepted with $1 - P_B$ where P_B is the blocking intercept. If $F[T]$ represents the time spent in the messaging facility, the throughput α of the arrangement is given by

$$\alpha = \frac{N}{F[T] + \frac{1}{[(1-P_B)\lambda]}} \quad (1)$$

$$\text{Where } F[T] = \frac{N}{\alpha} - \frac{1}{(1-P_B)\lambda}$$

If $F[L]$ represents the quantity of messages inside the service facility (queue size) instantly after the arbitrary slot limit, from the Little Theorem

$$F(L) = \gamma F(T) = N - \frac{\alpha}{\lambda(1-P_B)} \quad (2)$$

Hence $\alpha = (1 - P_B)\lambda(N - F[T])$

Consider the Markov chain of the quantity of messages in the service facility instantly after service achievement. Let be the probability that the messages in service k are immediately after the completion of a message in service, where $k = 0, 1, 2, \dots, K-1$, then, following the set of equations

$$\pi_k = \sum_{j=0}^{k+1} \pi_j P_{jk} \quad 0 \leq k \leq K-1 \quad (3)$$

$$\sum_{j=0}^{k+1} \pi_k = 1 \quad (4)$$

Where

$$P_{0,k} = \frac{1}{1-(1-\lambda)^N} \sum_{j=0}^{k+1} \binom{N}{j} \lambda^j (1-\lambda)^{N-j}$$

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$$\times \sum_{\ell=1}^{\infty} b(\ell) \binom{N-j}{k-j+1} [(1-\lambda)^\ell]^{N-k-1} [1-(1-\lambda)^\ell]^{k-j+1}$$

$$0 \leq k \leq K-2 \quad (5)$$

$$P_{0,k-1} = \frac{1}{1-(1-\lambda)^N} \sum_{j=1}^{k-1} \binom{N}{j} \lambda^j (1-\lambda)^{N-j}$$

$$\times \sum_{k=K-j}^{N-j} \sum_{\ell=1}^{\infty} b(\ell) \binom{N-j}{k} [(1-\lambda)^\ell]^{N-k-j} [1-(1-\lambda)^\ell]^k$$

$$+ \frac{1}{1-(1-\lambda)^N} \sum_{j=k}^k \binom{N}{j} \lambda^j (1-\lambda)^{N-j}$$

$$\times \sum_{k=0}^{N-k} \sum_{\ell=1}^{\infty} b(\ell) \binom{N-k}{k} [(1-\lambda)^\ell]^{N-K-k} [1-(1-\lambda)^\ell]^k$$

(6)

Queue size and enduring repair time:

The dual distribution of the queue size L and the enduring service time X instantly after a random slot edge is defined by

$$\prod_k(\ell) = Prob[L = k, X = \ell] \quad (7)$$

$$1 \leq k \leq K, \ell \geq 1$$

$$\prod_k^*(\ell) = \sum_{\ell=1}^{\infty} \prod_k(\ell) u^\ell \quad (8)$$

Then the following set of equations for P₀ and $\prod_k(\ell)$ are given by

$$P_0 = P_0(1-\lambda)^N + \prod_1(1)(1-\lambda)^{N-1} \quad (9)$$

$$\prod_k(\ell) = P_0 \binom{N}{k} \lambda^k (1-\lambda)^{N-k} b(\ell)$$

$$+ \sum_{j=1}^k \prod_j(\ell+1) \binom{N-j}{k-j} \lambda^{k-j} (1-\lambda)^{N-k}$$

$$+ \sum_{j=1}^{k+1} \prod_j(1) \binom{N-j}{k-j+1} \lambda^{k-j+1} (1-\lambda)^{N-k-1} b(\ell) \quad (10)$$

$$\prod_{k-1}(\ell) = P_0 \binom{N}{K-1} \lambda^{K-1} (1-\lambda)^{N-K+1} b(\ell)$$

$$+ \sum_{j=1}^{K-1} \prod_j(\ell+1) \binom{N-j}{K-j-1} \lambda^{K-j-1} (1-\lambda)^{N-K+1}$$

$$+ \sum_{j=1}^k \prod_j(1) \sum_{k=K-j}^{N-j} \binom{N-j}{k} \lambda^k (1-\lambda)^{N-K+1} b(\ell), \ell \geq 1 \quad (11)$$

Queue range at a random time:

The distribution of the queue size L instantly after a random slot edge is given P₀ and

$$Prob[L = k] = P_k = \prod_k^*(1) \quad 1 \leq k \leq K \quad (12)$$

Therefore,

$$\prod_1^*(u) = \frac{(1-\lambda)^{N-1} \prod_1(1) u \{B(u) - B[(1-\lambda)^{N-1}]\}}{B[(1-\lambda)^{N-1}][u - (1-\lambda)^{N-1}]} \quad (13)$$

Where

$$(1-\lambda)^{N-1} \prod_1(1) = P_0 [1 - (1-\lambda)^N] \quad (14)$$

$$\prod_1^*(1) = \frac{(1-\lambda)^{N-1} \prod_1(1) \{B(u) - B[(1-\lambda)^{N-1}]\}}{B[(1-\lambda)^{N-1}][u - (1-\lambda)^{N-1}]} \quad (15)$$



Uncompleted work and waiting time:

The PGF $U(u)$ for the uncompleted work U in the service capability instantly after an random slot edge is given by

$$U(u) = P_0 + \sum_{k=1}^K \Pi_k^*(u)[B(u)]^{k-1} \quad (16)$$

The mean uncompleted work is

$$E[U] = b \sum_{k=2}^k (k-1)P_k + \sum_{k=1}^k \Pi_k^{*(1)}(1) \quad (17)$$

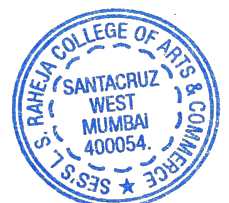
Now the PGF $W(u)$ for the waiting time W is accepted in an FCFS system.

$$\begin{aligned} \alpha W(u) = P_0 & \left[\sum_{j=1}^k \binom{N}{j} \lambda^j (1-\lambda)^{N-j} \sum_{m=1}^j [B(u)]^{m-1} \right. \\ & \left. + \sum_{j=k+1}^N \binom{N}{j} \lambda^j (1-\lambda)^{N-j} \sum_{m=1}^K [B(u)]^{m-1} \right] \\ & + \sum_{k=1}^{k-1} \sum_{\ell=1}^{\infty} \Pi_k(\ell) \\ & \left[\sum_{j=1}^{K-k} \binom{N-k}{j} \lambda^j (1-\lambda)^{N-k-j} u^{\ell-1} \sum_{m=1}^j [B(u)]^{k+m-2} \right] \\ & + \sum_{j=K-k+1}^{K-k} \binom{N-k}{j} \lambda^j (1-\lambda)^{N-k-j} u^{\ell-1} \sum_{m=1}^{K-k} [B(u)]^{k+m-2} \quad (18) \end{aligned}$$

Multiple vacation models: Here in Geo/G/1/K/N arrangement with various service vacations again assuming the late arrival model with partial acceptance and delayed replacement rule. After each slot limit the queue size L was observed alone which is not a Markov chain, the combined method of queue size L and the combined service time X or queue size L and the enduring service time X_+ practical after each slot.

Queue size at service end times: Let be the probability that the k messages in the service facility are immediately after completion of the service, where $k = 0, 1, 2, \dots, K-1$. The Markov chain before the number of messages in the service facility instantly after service is complete. Markov points for the queue size immediately after those slot boundaries at which either a setup or a service is completed. Let the probability that a Markov points is a setup end time and that there are k messages in the system instantly after that time, where $k = 1, 2, \dots, K$. Let the probability that a Markov point is a service end time and that there are k messages in the system instantly after that time, the set of possibility is given by

$$\begin{aligned} P_{0k} &= \frac{1}{1 - V[(1-\lambda)^N]} \sum_{j=1}^{k+1} \sum_{\ell=1}^{\infty} v(\ell) \binom{N}{j} [(1-\lambda)^\ell]^{N-j} [1 - (1-\lambda)^\ell]^j \times \\ & \sum_{\ell=1}^{\infty} b(\ell) \binom{N-j}{k-j+1} [(1-\lambda)^\ell]^{N-j} [1 - (1-\lambda)^\ell]^j \\ & \quad 0 \leq k \leq K-2 \quad (19) \\ P_{0,k-1} &= \frac{1}{1 - V[(1-\lambda)^N]} \sum_{j=1}^{k-1} \sum_{\ell=1}^{\infty} v(\ell) \binom{N}{j} \lambda^j (1-\lambda)^{N-j} \\ & \times \sum_{k=K-j}^{N-j} \sum_{\ell=1}^{\infty} \binom{N-j}{k} [(1-\lambda)^\ell]^{N-j-k} [1 - (1-\lambda)^\ell]^k \end{aligned}$$



$$\begin{aligned}
 &+ \frac{1}{1-v[(1-\lambda)^N]} \sum_{j=k}^{N-j} \sum_{\ell}^{\infty} v(\ell) \binom{N}{j} \lambda^j (1-\lambda)^{N-j} \\
 &\quad \times \sum_{k=0}^{N-k} \sum_{\ell=1}^{\infty} \binom{N-j}{k} [(1-\lambda)^\ell]^{N-j-k} [1-(1-\lambda)^\ell]^k \quad (20)
 \end{aligned}$$

Unfinished work and waiting time: Now suppose that are known, because the whole coefficients are known. PGF U (u) for unfinished work U is a service facility given immediately by an arbitrary slot limit.

$$\begin{aligned}
 U(u) &= \sum_{k=0}^K \Omega_k(1)[B(u)]^k \\
 &+ \sum_{k=1}^K \prod_{k=1}^k (u)[B(u)]^{k-1} \quad (21)
 \end{aligned}$$

Queue size and Elapsed vacation service time: Consider the joint probability distribution is given by

$$\begin{aligned}
 Q_k(\ell) &= Prob[\zeta = 0, L = k, V = \ell] \\
 &0 \leq k \leq K, \ell \geq 0 \\
 P_k(\ell) &= Prob[\zeta = 0, L = k, X = \ell] \\
 &1 \leq k \leq K, \ell \geq 0
 \end{aligned}$$

The queue size L and the elapsed vacation time V or the elapsed service time X immediately after a random slot edge,

$$\begin{aligned}
 Q_0(0) &= \sum_{\ell=1}^{\infty} Q_0(\ell-1)v(\ell)(1-\lambda)^N \\
 &+ \sum_{\ell=1}^{\infty} P_1(\ell-1)b(\ell)(1-\lambda)^{N-1} \quad (22) \\
 Q_k(0) &= 0 \quad 1 \leq k \leq K
 \end{aligned}$$

$$\begin{aligned}
 Q_k(\ell) &= \sum_{j=0}^k Q_j(\ell-1)[1-v(1)] \binom{N-j}{k-j} \lambda^{k-j} (1-\lambda)^{N-k} \\
 &0 \leq k \leq K, \ell \geq 1 \quad (23)
 \end{aligned}$$

$$Q_k(\ell) = \sum_{j=0}^k Q_j(\ell-1)[1-v(1)] \binom{N-j}{k} \lambda^k (1-\lambda)^{N-j-k}, \ell \geq 1 \quad (24)$$

Multiple holiday model: The arrangement with multiple server holidays, the probability that K messages arrive during the holiday and the probability that K, where K = 0, 1, 2...

$$\sum_{k=0}^{\infty} f_k z^k = V(1-\lambda + \lambda z) \quad (30)$$

Where V (u) is the PGF for V at the time of vacation. The length of the elapsed vacation time (V-) and the length of the enduring vacation time is given by the dual distribution for the number of incoming messages during (V+).

$$\begin{aligned}
 \phi(u) &= F \left[\binom{V}{n} \lambda^n (1-\lambda)^{V-n} u^{V+} \right] \\
 &= \frac{u}{\lambda F[V]} \left[V(u) \left(\frac{\lambda}{u-1+\lambda} \right)^{n+1} \right] \\
 &- \sum_{\ell=0}^n f_{\ell} \left(\frac{\lambda}{u-1+\lambda} \right)^{n-\ell+1} \quad (25)
 \end{aligned}$$

The size of the queue instantly after the discharge or service is complete, the order of the queue size constitutes a Markov chain. Let q_k be the probability that an observation point is instantly after the completion of a holiday and that there are K messages in the system instantly after the time where K = 0, 1, 2... k. Similarly, let us consider the probability that an observation point is instantly after the completion of a service and instantly after that time there are k messages in the system, where k = 0,1,2... k - 1, then the equation for the following set. The queue size distribution instantly after a random slot edge, the system state



during a vacation by keeping track of a joint process for the queue size, the enduring service time and the elapsed vacation time at each slot edge.

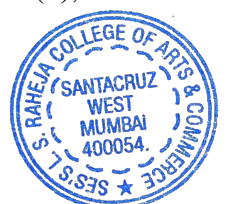
Pure decrementing service system: In a pure decrementing service system with multiple vacations, if there is at least one message in the system when the server returns from a vacation, the service is started and continued until the number of messages in the system decreases to one less than that found at the end of the vacation. If there are no messages in the queue at the end of a vacation, the server takes another vacation, and repeats vacations until at least one message is found in the queue at the end of a vacation. Therefore, once, a service is started after a vacation, the length of a service period is identical with the length of a busy stage started with the service to a only message in a system without vacation.

2. CONCLUSION:

The Dual distribution of the queue size and the elapsed service time immediately after an arbitrary slot boundary, its marginal distribution gives the distribution of the queue size immediately after an arbitrary slot boundary the joint distribution of the queue size and the remaining service time, which enables us to obtain the distribution of the unfinished work immediately after an arbitrary slot boundary.

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